PIONEERS OF PROGRESS

MEN OF SCIENCE

EDITED BY S. CHAPMAN, M.A., D.Sc., F.R.S.

THE COPERNICUS OF ANTIQUITY

(ARISTARCHUS OF SAMOS)

BY

SIR THOMAS HEATH

K.C.B., K.C.V.O., F.R.S.; Sc.D., CAMB.; HON. D.Sc., OXFORD

LONDON:

SOCIETY FOR PROMOTING CHRISTIAN KNOWLEDGE

NEW YORK: THE MACMILLAN COMPANY

1920

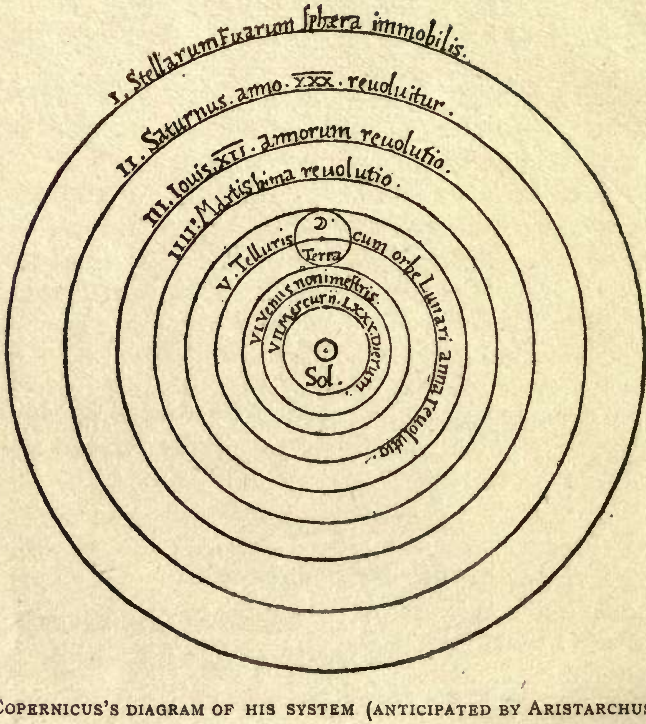
CONTENTS

PART I

GREEK ASTRONOMY TO ARISTARCHUS

PART II

ARISTARCHUS OF SAMOS



COPERNICUS'S DIAGRAM OF HIS SYSTEM (ANTICIPATED BY ARISTARCHUS).

PART I.

GREEK ASTRONOMY TO ARISTARCHUS.

THE title-page of this book necessarily bears the name of one man; but the reader will find in its pages the story, or part of the story, of many other Pioneers of Progress. The crowning achievement of anticipating the hypothesis of Copernicus belongs to Aristarchus of Samos alone; but to see it in its proper setting it is necessary to have followed in the footsteps of the earlier pioneers who, by one bold speculation after another, brought the solution of the problem nearer, though no one before Aristarchus actually hit upon the truth. This is why the writer has thought it useful to prefix to his account of Aristarchus a short sketch of the history of the development of astronomy in Greece down to Aristarchus's time, which is indeed the most fascinating portion of the story of Greek astronomy.

The extraordinary advance in astronomy made by the Greeks in a period of little more than three centuries is a worthy parallel to the rapid development, in their hands, of pure geometry, which, created by them as a theoretical science about the same time, had by the time of Aristarchus covered the ground of the Elements (including solid geometry and the geometry of the sphere), had established the main properties of the three conic sections, had solved problems which were beyond the geometry of the straight line and circle, and finally, before the end of the third century B.C., had been carried to its highest perfection by the genius of Archimedes, who measured the areas of curves and the surfaces and volumes of curved surfaces by geometrical methods practically anticipating the integral calculus.

To understand how all this was possible we have to remember that the Greeks, pre-eminently among all the nations of the world, possessed just those gifts which are essential to the initiation and development of philosophy and science. They had in the first place a remarkable power of accurate observation; and to this were added clearness of intellect to see things as they are, a passionate love of knowledge for its own sake, and a genius for speculation which stands unrivalled to this day. Nothing that is perceptible to the senses seems to have escaped them; and when the apparent facts had been accurately ascertained, they wanted to know the *why* and the *wherefore*, never resting satisfied until they had given a rational explanation, or what seemed to them to be such, of the phenomena observed. Observation or experiment and theory went hand in hand. So it was that they developed such subjects as medicine and astronomy. In astronomy their guiding principle was, in their own expressive words, to "save the phenomena". This meant that, as more and more facts became known, their theories were continually revised to fit them.

It would be easy to multiply instances; it must suffice in this place to mention one, which illustrates not only the certainty with which the Greeks detected the occurrence of even the rarest phenomena, but also the persistence with which they sought for the true explanation.

Cleomedes (second century A.D.) mentions that there were stories of extraordinary eclipses which "the more ancient of the mathematicians" had vainly tried to explain; the supposed "paradoxical" case was that in which, while the sun seems to be still above the western horizon, the *eclipsed* moon is seen to rise in the east. The phenomenon appeared to be inconsistent with the explanation of lunar eclipses by the entry of the moon into the earth's shadow; how could this be if both bodies were above the horizon at the same time? The "more ancient" mathematicians essayed a geometrical explanation; they tried to argue that it was possible that a spectator standing on an *eminence* of the spherical earth might see along the generators of a *cone*, i.e. a little downwards on all sides instead of merely in the plane of the horizon, and so might see both the sun and the moon even when the latter was in the earth's shadow. Cleomedes denies this and prefers to regard the whole story of such cases as a fiction designed merely for the purpose of plaguing astronomers and philosophers; no Chaldæan, he says, no Egyptian, and no mathematician or philosopher has recorded such a case. But the phenomenon is possible, and it is certain that it had been observed in Greece and that the Greek astronomers did not rest until they had found out the solution of the puzzle; for Cleomedes himself gives the explanation, namely that the phenomenon is due to atmospheric refraction. Observing that such cases of atmospheric refraction were especially noticeable in the neighbourhood of the Black Sea, Cleomedes goes on to say that it is possible that the visual rays going out from our eyes are refracted through falling on wet and damp air, and so reach the sun although it is already below the horizon; and he compares the well-known experiment of the ring at the bottom of a jug, where the ring, just out of sight when the jug is empty, is brought into view when water is poured in.

The genius of the race being what it was, the Greeks must from the earliest times have been in the habit of scanning the heavens, and, as might be expected, we find the beginnings of astronomical knowledge in the earliest Greek literature.

In the Homeric poems and in Hesiod the earth is a flat circular disc; round this disc runs the river Oceanus, encircling the earth and flowing back into itself. The flat earth has above it the vault of heaven, like a sort of hemispherical dome exactly covering it; this vault remains for ever in one position; the sun, moon and stars move round under it, rising from Oceanus in the east and plunging into it again in the west.

Homer mentions, in addition to the sun and moon, the Morning Star, the Evening Star, the Pleiades, the Hyades, Orion, the Great Bear ("which is also called by the name of the Wain"), Sirius, the late-setting Boötes (the ploughman driving the Wain), i.e. Arcturus, as it was first called by Hesiod. Of the Great Bear Homer says that it turns round on the same spot and watches Orion; it alone is without lot in Oceanus's bath (i.e. it never sets). With regard to the last statement it is to be noted that some of the principal stars of the Great Bear do now set in the Mediterranean, e.g. in places further south than Rhodes (lat. 36°), γ, the hind foot, and η, the tip of the tail, and at Alexandria all the seven stars except α, the head. It might be supposed that here was a case of Homer "nodding". But no; the old poet was perfectly right; the difference between the facts as observed by him and as seen by us respectively is due to the Precession of the Equinoxes, the gradual movement of the fixed stars themselves about the pole of the ecliptic, which was discovered by Hipparchus (second century B.C.). We know from the original writings of the Greek astronomers that in Eudoxus's time (say 380 B.C.) the whole of the Great Bear remained always well above the horizon, while in the time of Proclus (say A.D. 460) the Great Bear "grazed" the horizon.

In Homer astronomical phenomena are only vaguely used for such purposes as fixing localities or marking times of day or night. Sometimes constellations are used in giving sailing directions, as when Calypso directs Odysseus to sail in such a way as always to keep the Great Bear on his left.

Hesiod mentions practically the same stars as Homer, but makes more use of celestial phenomena for determining times and seasons. For example, he marked the time for sowing at the beginning of winter by the setting of the Pleiades in the early twilight, or again by the early setting of the Hyades or Orion, which means the 3rd, 7th, or 15th November in the Julian calendar according to the particular stars taken; the time for harvest he fixed by the early rising of the Pleiades (19th May), threshing time by the early rising of Orion (9th July), vintage time by the early rising of Arcturus (18th September), and so on. Hesiod makes spring begin sixty days after the winter solstice, and the early summer fifty days after the summer solstice. Thus he knew about the solstices, though he says nothing of the equinoxes. He had an approximate notion of the moon's period, which he put at thirty days.

But this use of astronomical facts for the purpose of determining times and seasons or deducing weather indications is a very different thing from the science of astronomy, which seeks to explain the heavenly phenomena and their causes. The history of this science, as of Greek philosophy in general, begins with Thales.

The Ionian Greeks were in the most favourable position for initiating philosophy. Foremost among the Greeks in the love of adventure and the instinct of new discovery (as is shown by their leaving their homes to found settlements in distant lands), and fired, like all Greeks, with a passion for knowledge, they needed little impulse to set them on the road of independent thought and speculation. This impulse was furnished by their contact with two ancient civilisations, the Egyptian and the Babylonian. Acquiring from them certain elementary facts and rules in mathematics and astronomy which had been handed down through the priesthood from remote antiquity, they built upon them the foundation of the science, as distinct from the mere routine, of the subjects in question.

THALES.

Thales of Miletus (about 624-547 B.C.) was a man of extraordinary versatility; philosopher, mathematician, astronomer, statesman, engineer, and man of business, he was declared one of the Seven Wise Men in 582-581 B.C. His propensity to star-gazing is attested by the story of his having fallen into a well while watching the stars, insomuch that (as Plato has it) he was rallied by a clever and pretty maidservant from Thrace for being so "eager to know what goes on in the heavens when he could not see what was in front of him, nay at his very feet".

Thales's claim to a place in the history of scientific astronomy rests on one achievement attributed to him, that of predicting an eclipse of the sun. The evidence for this is fairly conclusive, though the accounts of it differ slightly. Eudemus, the pupil of Aristotle, who wrote histories of Greek geometry and astronomy, is quoted by three different Greek writers as the authority for the story. But there is testimony much earlier than this. Herodotus, speaking of a war between the Lydians and the Medes, says that, "when in the sixth year they encountered one another, it fell out that, after they had joined battle, the day suddenly turned into night. Now that this change of day into night would occur was foretold to the Ionians by Thales of Miletus, who fixed as the limit of time this very year in which the change took place." Moreover Xenophanes, who was born some twenty-three years before Thales's death, is said to have lauded Thales's achievement; this would amount to almost contemporary evidence.

Could Thales have known the cause of solar eclipses? Aëtius (A.D. 100), the author of an epitome of an older collection of the opinions of philosophers, says that Thales was the first to declare that the sun is eclipsed when the moon comes in a direct line below it, the image of the moon then appearing on the sun's disc as on a mirror; he also associates Thales with Anaxagoras, Plato, Aristotle, and the Stoics as holding that the moon is eclipsed by reason of its falling into the shadow made by the earth when the earth is between the sun and the moon. But, as regards the eclipse of the moon, Thales could not have given this explanation, because he held that the earth (which he presumably regarded as a flat disc) floated on the water like a log. And if he had given the true explanation of a solar eclipse, it is impossible that all the succeeding Ionian philosophers should have exhausted their imaginations in other fanciful explanations such as we find recorded.

The key to the puzzle may be afforded by the passage of Herodotus according to which the prediction was a rough one, only specifying that the eclipse would occur within a certain year. The prediction was probably one of the same kind as had long been made by the Chaldæans. The Chaldæans, no doubt as the result of observations continued through many centuries, had discovered the period of 223 lunations after which lunar eclipses recur. (This method would very often fail for solar eclipses because no account was taken of parallax; and Assyrian cuneiform inscriptions record failures as well as successful predictions.) Thales, then, probably learnt about the period of 223 lunations either in Egypt or more directly through Lydia, which was an outpost of Assyrio-Babylonian culture. If there happened to be a number of *possible* solar eclipses in the year which (according to Herodotus) Thales fixed for the eclipse, he was, in using the Chaldæan rule, not taking an undue risk; but it was great luck that the eclipse should have been total. It seems practically certain that the eclipse in question was that of the (Julian) 28th May, 585.

Thales, as we have seen, made the earth a circular or cylindrical disc floating on the water like a log or a cork and, so far as we can judge of his general conception of the universe, he would appear to have regarded it as a mass of water (that on which the earth floats) with the heavens encircling it in the form of a hemisphere and also bounded by the primeval water. This view of the world has been compared with that found in ancient Egyptian papyri. In the beginning existed the *Nū*, a primordial liquid mass in the limitless depths of which floated the germs of things. When the sun began to shine, the earth was flattened out and the water separated into two masses. The one gave rise to the rivers and the ocean, the other, suspended above, formed the vault of heaven, the waters above, on which the stars and the gods, borne by an eternal current, began to float. The sun, standing upright in his sacred barque which had endured for millions of years, glides slowly, conducted by an army of secondary gods, the planets and the fixed stars. The assumption of an upper and lower ocean is also old Babylonian (cf. the division in Genesis I. 7 of the waters which were under the firmament from the waters which were above the firmament).

It would follow from Thales's general view of the universe that the sun, moon and stars did not, between their setting and rising again, continue their circular path below the earth but (as with Anaximenes later) moved laterally round the earth.

Thales's further contributions to observational astronomy may be shortly stated. He wrote two works *On the solstice* and *On the equinox*, and he is said by Eudemus to have discovered that "the period of the sun with respect to the solstices is not always the same," which probably means that he discovered the inequality of the four astronomical seasons. His division of the year into 365 days he probably learnt from the Egyptians. He said of the Hyades that there are two, one north and the other south. He observed the Little Bear and used it as a means of finding the pole; he advised the Greeks to follow the Phœnician plan of sailing by the Little Bear in preference to their own habit of steering by the Great Bear.

Limited as the certain contributions of Thales to astronomy are, it became the habit of the Greek *Doxographi*, or retailers of the opinions of philosophers, to attribute to Thales, in common with other astronomers in each case, a number of discoveries which were not made till later. The following is a list, with (in brackets) the names of the astronomers to whom the respective discoveries may with most certainty be assigned: (1) the fact that the moon takes its light from the sun (Anaxagoras), (2) the sphericity of the earth (Pythagoras), (3) the division of the heavenly sphere into five zones (Pythagoras and Parmenides), (4) the obliquity of the ecliptic (Oenopides of Chios), and (5) the estimate of the sun's apparent diameter as $\frac{1}{720}$th of the sun's circle (Aristarchus of Samos).

ANAXIMANDER.

Anaximander (about 611-547 B.C.), a contemporary and fellow-citizen of Thales, was a remarkably original thinker. He was the first Greek philosopher who ventured to put forward his views in a formal written treatise. This was a work *About Nature* and was not given to the world till he was about sixty-four years old. His originality is illustrated by his theory of evolution. According to him animals first arose from slime evaporated by the sun; they lived in the sea and had prickly coverings; men at first resembled fishes.

But his astronomical views were not less remarkable. Anaximander boldly maintained that the earth is in the centre of the universe, suspended freely and without support, whereas Thales regarded it as resting on the water and Anaximenes as supported by the air. It remains in its position, said Anaximander, because it is at an equal distance from all the rest of the heavenly bodies. The earth was, according to him, cylinder-shaped, round "like a stone pillar"; one of its two plane faces is that on which we stand; its depth is one-third of its breadth.

Anaximander postulated as his first principle, not water (like Thales) or any of the elements, but the Infinite; this was a substance, not further defined, from which all the heavens and the worlds in them were produced; according to him the worlds themselves were infinite in number, and there were always some worlds coming into being and others passing away *ad infinitum*. The origin of the stars, and their nature, he explained as follows. "That which is capable of begetting the hot and the cold out of the eternal was separated off during the coming into being of our world, and from the flame thus produced a sort of sphere was made which grew round the air about the earth as the bark round the tree; then this sphere was torn off and became enclosed in certain circles or rings, and thus were formed the sun, the moon and the stars." "The stars are produced as a circle of fire, separated off from the fire in the universe and enclosed by air. They have as vents certain pipe-shaped passages at which the stars are seen." "The stars are compressed portions of air, in the shape of wheels filled with fire, and they emit flames at some point from small openings." "The stars are borne round by the circles in which they are enclosed." "The sun is a circle twenty-eight times (*v. l.* 27 times) the size of the earth; it is like a wheel of a chariot the rim of which is hollow and full of fire and lets the fire shine out at a certain point in it through an opening like the tube of a blow-pipe; such is the sun." "The sun is equal to the earth." "The eclipses of the sun occur through the opening by which the fire finds vent being shut up." "The moon is a circle nineteen times the size of the earth; it is similar to a chariot-wheel the rim of which is hollow and full of fire like the circle of the sun, and it is placed obliquely like the other; it has one vent like the tube of a blow-pipe; the eclipses of the moon depend on the turnings of the wheel." "The moon is eclipsed when the opening in the rim of the wheel is stopped up." "The moon appears sometimes as waxing, sometimes as waning, to an extent corresponding to the closing or opening of the passages." "The sun is placed highest of all, after it the moon, and under them the fixed stars and the planets."

It has been pointed out that the idea of the formation of tubes of compressed air within which the fire of each star is shut up except for the one opening through which the flame shows (like a gas-jet, as it were) is not unlike Laplace's hypothesis with reference to the origin of Saturn's rings. In any case it is a sufficiently original conception.

When Anaximander says that the hoops carrying the sun and moon "lie obliquely," this is no doubt an attempt to explain, in addition to the daily rotation, the annual movement of the sun and the monthly movement of the moon.

We have here too the first speculation about the sizes and distances of the heavenly bodies. The sun is as large as the earth. The ambiguity between the estimates of the size of the sun's circle as twenty-seven or twenty-eight times the size of the earth suggests that it is a question between taking the inner and outer circumferences of the sun's ring respectively, and a similar ambiguity may account for the circle of the moon being stated to be nineteen times, not eighteen times, the size of the earth. No estimate is given of the distance of the planets from the earth, but as, according to Anaximander, they are nearer to the earth than the sun and moon are, it is possible that, if a figure had been stated, it would have been nine times the size of the earth, in which case we should have had the numbers 9, 18, 27, three terms in arithmetical progression and all of them multiples of 9, the square of 3. It seems probable that these figures were not arrived at by any calculation based on geometrical considerations, but that we have here merely an illustration of the ancient cult of the sacred numbers 3 and 9. Three is the sacred number in Homer, 9 in Theognis. The cult of 3 and its multiples 9 and 27 is found among the Aryans, then among the Finns and Tartars and then again among the Etruscans. Therefore Anaximander's figures probably say little more than what the Indians tell us, namely, that three Vishnu-steps reach from earth to heaven.

Anaximander is said to have been the first to discover the *gnomon* (or sun-dial with a vertical needle). This is, however, incorrect, for Herodotus says that the Greeks learnt the use of the *gnomon* and the *polos* from the Babylonians. Anaximander may have been the first to introduce the gnomon into Greece. He is said to have set it up in Sparta and to have shown on it "the solstices, the times, the seasons, and the equinox".

But Anaximander has another title to fame. He was the first who ventured to draw a map of the inhabited earth. The Egyptians indeed had drawn maps before, but only of special districts. Anaximander boldly planned out the whole world with "the circumference of the earth and of the sea". Hecataeus, a much-travelled man, is said to have corrected Anaximander's map so that it became the object of general admiration.

ANAXIMENES.

With Anaximenes of Miletus (about 585-528/4 B.C.) the earth is still flat like a table, but, instead of being suspended freely without support as with Anaximander, it is supported by the air, riding on it as it were. The sun, moon and stars are all made of fire and (like the earth) they ride on the air because of their breadth. The sun is flat like a leaf. Anaximenes also held that the stars are fastened on a crystal sphere like nails or studs. It seems clear therefore that by the stars which "ride on the air because of their breadth" he meant the planets only. A like apparent inconsistency applies to the motion of the stars. If the stars are fixed in the crystal sphere like nails, they must be carried round complete circles by the revolution of the sphere about a diameter.

Yet Anaximenes also said that the stars do not move or revolve *under* the earth as some suppose, but *round* the earth, just as a cap can be turned round on the head. The sun is hidden from sight, not because it is under the earth, but because it is covered by the higher parts of the earth and because its distance from us is greater. Aristotle adds the detail that the sun is carried round the northern portion of the earth and produces night because the earth is lofty towards the north. We must again conclude that the stars which, like the sun and moon, move laterally round the earth between their setting and rising again are the planets, as distinct from the fixed stars. It would therefore seem that Anaximenes was the first to distinguish the planets from the fixed stars in respect of their irregular movements. He improved on Anaximander in that he relegated the fixed stars to the region most distant from the earth.

Anaximenes was also original in holding that, in the region occupied by the stars, bodies of an earthy nature are carried round along with them. The object of these invisible bodies of an earthy nature carried round along with the stars is clearly to explain the eclipses and phases of the moon. It was doubtless this conception which, in the hands of Anaxagoras and others, ultimately led to the true explanation of eclipses.

The one feature of Anaximenes's system which was destined to an enduring triumph was the conception of the stars being fixed on a crystal sphere as in a rigid frame. This really remained the fundamental principle in all astronomy down to Copernicus.

PYTHAGORAS.

With Pythagoras and the Pythagoreans we come to a different order of things. Pythagoras, born at Samos about 572 B.C., is undoubtedly one of the greatest names in the history of science. He was a mathematician of brilliant achievements; he was also the inventor of the science of acoustics, an astronomer of great originality, a theologian and moral reformer, and the founder of a brotherhood which admits comparison with the orders of mediæval chivalry. Perhaps his most epoch-making discovery was that of the dependence of musical tones on numerical proportions, the octave representing the proportion of 2 : 1 in length of string at the same tension, the fifth 3 : 2, and the fourth 4 : 3. Mathematicians know him as the reputed discoverer of the famous theorem about the square on the hypotenuse of a right-angled triangle (= Euclid I. 47); but he was also the first to make geometry a part of a liberal education and to explore its first principles (definitions, etc.).

Pythagoras is said to have been the first to maintain that the earth is spherical in shape; on what ground, is uncertain. One suggestion is that he may have argued from the roundness of the shadow cast by the earth in the eclipses of the moon; but Anaxagoras was the first to give the true explanation of such eclipses. Probably Pythagoras attributed spherical shape to the earth for the mathematical or mathematico-æsthetical reason that the sphere is the most beautiful of all solid figures. It is probable too, and for the same reason, that Pythagoras gave the same spherical shape to the sun and moon, and even to the stars, in which case the way lay open for the discovery of the true cause of eclipses and of the phases of the moon. Pythagoras is also said to have distinguished five zones in the earth. It is true that the first declaration that the earth is spherical and that it has five zones is alternatively attributed to Parmenides (born perhaps about 516 or 514 B.C.), on the good authority of Theophrastus. It is possible that, although Pythagoras was the real author of these views, Parmenides was the first to state them in public.

Pythagoras regarded the universe as living, intelligent, spherical, enclosing the earth at the centre, and rotating about an axis passing through the centre of the earth, the earth remaining at rest.

He is said to have been the first to observe that the planets have an independent motion of their own in a direction opposite to that of the fixed stars, i.e. the daily rotation. Alternatively with Parmenides he is said to have been the first to recognise that the Morning and the Evening Stars are one and the same. Pythagoras is hardly likely to have known this as the result of observations of his own; he may have learnt it from Egypt or Chaldæa along with other facts about the planets.

PARMENIDES.

We have seen that certain views are alternatively ascribed to Pythagoras and Parmenides. The system of Parmenides was in fact a kind of blend of the theories of Pythagoras and Anaximander. In giving the earth spherical form with five zones he agreed with Pythagoras. Pythagoras, however, made the spherical universe rotate about an axis through the centre of the earth; this implied that the universe is itself limited, but that something exists round it, and in fact that beyond the finite rotating sphere there is limitless void or empty space. Parmenides, on the other hand, denied the existence of the infinite void and was therefore obliged to make his finite sphere motionless and to hold that its apparent rotation is only an illusion.

In other portions of his system Parmenides followed the lead of Anaximander. Like Anaximander (and Democritus later) he argued that the earth remains in the centre because, being equidistant from all points on the sphere of the universe, it is in equilibrium and there is no more reason why it should tend to move in one direction than in another. Parmenides also had a system of wreaths or bands round the sphere of the universe which contained the sun, the moon and the stars; the wreaths remind us of the hoops of Anaximander, but their nature is different. The wreaths, according to the most probable interpretation of the texts, are, starting from the outside, (1) a solid envelope like a wall; (2) a band of fire (the æther-fire); (3) mixed bands, made up of light and darkness in combination, which exhibit the phenomenon of "fire shining out here and there," these mixed bands including the Milky Way as well as the sun, moon and planets; (4) a band of fire, the inner side of which is our atmosphere, touching the earth. Except that Parmenides placed the Morning Star first in the æther and therefore above the sun, he did not apparently differ from Anaximander's view of the relative distances of the heavenly bodies, according to which both the planets and the other stars are all placed below the sun and moon.

Two lines from Parmenides's poem have been quoted to show that he declared that the moon is illuminated by the sun. The first line speaks of the moon as "a night-shining foreign light wandering round the earth"; but, even if the line is genuine, "foreign" need not mean "borrowed". The other line speaks of the moon as "always fixing its gaze on the sun"; but, though this states an observed fact, it is far from explaining the cause. We have, moreover, positive evidence against the attribution of the discovery of the opacity of the moon to Parmenides. It is part of the connected prose description of his system that the moon is a mixture of air and fire, and in other passages we are told that he held the moon to be of fire. Lastly, Plato speaks of "the fact which Anaxagoras *lately asserted*, that the moon has its light from the sun". It seems impossible that Plato would speak in such terms if the fact in question had been stated for the first time either by Parmenides or by the Pythagoreans.

ANAXAGORAS.

Anaxagoras, a man of science if ever there was one, was born at Clazomenae in the neighbourhood of Smyrna about 500 B.C. He neglected his possessions, which were considerable, in order to devote himself to science. Someone once asked him what was the object of being born, and he replied, "The investigation of sun, moon and heaven". He took up his abode at Athens, where he enjoyed the friendship of Pericles. When Pericles became unpopular shortly before the outbreak of the Peloponnesian war, he was attacked through his friends, and Anaxagoras was accused of impiety for declaring that the sun was a red-hot stone and the moon made of earth. One account says that he was fined and banished; another that he was imprisoned, and that it was intended to put him to death, but that Pericles obtained his release; he retired to Lampsacus, where he died at the age of seventy-two.

One epoch-making discovery belongs to him, namely, that the moon does not shine by its own light but receives its light from the sun: Plato, as we have seen, is one authority for this statement. Plutarch also in his *De facie in orbe lunae* says, "Now when our comrade in his discourse had expounded that proposition of Anaxagoras that 'the sun places the brightness in the moon,' he was greatly applauded".

This discovery enabled Anaxagoras to say that "the obscurations of the moon month by month were due to its following the course of the sun by which it is illuminated, and the eclipses of the moon were caused by its falling within the shadow of the earth which then comes between the sun and the moon, while the eclipses of the sun were due to the interposition of the moon". Anaxagoras was therefore the first to give the true explanation of eclipses. As regards the phases of the moon, his explanation could only have been complete if he had known that the moon is spherical; in fact, however, he considered the earth (and doubtless the other heavenly bodies also) to be flat. To his true theory of eclipses Anaxagoras added the unnecessary assumption that the moon was sometimes eclipsed by other earthy bodies below the moon but invisible to us. In this latter assumption he followed the lead of Anaximenes. The other bodies in question were probably invented to explain why the eclipses of the moon are seen oftener than those of the sun.

Anaxagoras's cosmogony contained some fruitful ideas. According to him, the formation of the world began with a vortex set up, in a portion of the mixed mass in which "all things were together," by Mind. This rotatory movement began at one point and then gradually spread, taking in wider and wider circles. The first effect was to separate two great masses, one consisting of the rare, hot, light, dry, called the æther, and the other of the opposite categories and called air. The æther took the outer place, the air the inner. Out of the air were separated successively clouds, water, earth, and stones. The dense, the moist, the dark and cold, and all the heaviest things, collect in the centre as the result of the circular motion, and it is from these elements when consolidated that the earth is formed. But after this, "in consequence of the violence of the whirling motion, the surrounding fiery æther tore stones away from the earth and kindled them into stars". Anaxagoras conceived therefore the idea of a *centrifugal* force, as distinct from that of concentration brought about by the motion of the vortex, and he assumed a series of projections or "hurlings-off" of precisely the same kind as the theory of Kant and Laplace assumed for the formation of the solar system.

In other matters than the above Anaxagoras did not make much advance on the crude Ionian theories. "The sun is a red-hot mass or a stone on fire." "It is larger (or 'many times larger') than the Peloponnese." He considered that "the stars were originally carried round (laterally) like a dome, the pole which is always visible being thus vertically above the earth, and it was only afterwards that their course became inclined".

But he put forward a remarkable and original hypothesis to explain the Milky Way. He thought the sun to be smaller than the earth. Consequently, when the sun in its revolution passes below the earth, the shadow cast by the earth extends without limit. The trace of this shadow on the heavens is the Milky Way. The stars within this shadow are not interfered with by the light of the sun, and we therefore see them shining; those stars, on the other hand, which are outside the shadow are overpowered by the light of the sun which shines on them even during the night, so that we cannot see them. Aristotle easily disposes of this theory by observing that, the sun being much larger than the earth, and the distance of the stars from the earth being many times greater than the distance of the sun, the sun's shadow would form a cone with its vertex not very far from the earth, so that the shadow of the earth, which we call night, would not reach the stars at all.

EMPEDOCLES.

Empedocles of Agrigentum (about 494-434 B.C.) would hardly deserve mention for his astronomy alone, so crude were his views where they differed from those of his predecessors. The earth, according to Empedocles, is kept in its place by the swiftness of the revolution of the heaven, just as we may swing a cup with water in it round and round so that in some positions the top of the cup may even be turned downwards without the water escaping. Day and night he explained as follows. Within the crystal sphere to which the fixed stars are attached (as Anaximenes held), and filling it, is a sphere consisting of two hemispheres, one of which is wholly of fire and therefore light, while the other is a mixture of air with a little fire, which mixture is darkness or night. The revolution of these two hemispheres round the earth produces at each point on its surface the succession of day and night. Empedocles held the sun to be, not fire, but a reflection of fire similar to that which takes place from the surface of water, the fire of a whole hemisphere of the world being bent back from the earth, which is circular, and concentrated into the crystalline sun which is carried round by the motion of the fiery hemisphere.

Empedocles's one important scientific achievement was his theory that light travels and takes time to pass from one point to another. The theory is alluded to by Aristotle, who says that, according to Empedocles, the light from the sun reaches the intervening space before it reaches the eye or the earth; there was therefore a time when the ray was not yet seen, but was being transmitted through the medium.

THE PYTHAGOREANS.

We have seen that Pythagoras was the first to give spherical form to the earth and probably to the heavenly bodies generally, and to assign to the planets a revolution of their own in a sense opposite to that of the daily rotation of the fixed stars about the earth as centre.

But a much more remarkable development was to follow in the Pythagorean school. This was nothing less than the abandonment of the geocentric hypothesis and the reduction of the earth to the status of a planet like the others. The resulting system, known as the Pythagorean, is attributed (on the authority probably of Theophrastus) to Philolaus; but Diogenes Laertius and Aëtius refer to one Hicetas of Syracuse in this connection; Aristotle attributes the system to "the Pythagoreans". It is a partial anticipation of the theory of Copernicus but differs from it in that the earth and the planets do not revolve round the sun but about an assumed "central fire," and the sun itself as well as the moon does the same. There were thus eight heavenly bodies, in addition to the sphere of the fixed stars, all revolving about the central fire. The number of revolutions being thus increased to nine, the Pythagoreans postulated yet another, making ten. The tenth body they called the counter-earth, and its character and object will appear from the following general description of the system.

The universe is spherical in shape and finite in size. Outside it is infinite void, which enables the universe to breathe, as it were. At the centre is the central fire, the Hearth of the Universe, called by various names such as the Tower or Watch-tower of Zeus, the Throne of Zeus, the Mother of the Gods. In this central fire is located the governing principle, the force which directs the movement and activity of the universe. The outside boundary of the sphere is an envelope of fire; this is called Olympus, and in this region the elements are found in all their purity; below this is the universe. In the universe there revolve in circles round the central fire the following bodies: nearest to the central fire the counter-earth which always accompanies the earth, then the earth, then the moon, then the sun, next to the sun the five planets, and last of all, outside the orbits of the planets, the sphere of the fixed stars. The counter-earth, which accompanies the earth but revolves in a smaller orbit, is not seen by us because the hemisphere on which we live is turned away from the counter-earth. It follows that our hemisphere is always turned away from the central fire, that is, it faces outwards from the orbit towards Olympus (the analogy of the moon which always turns one side towards us may have suggested this); this involves a rotation of the earth about its axis completed in the same time as it takes the earth to complete a revolution about the central fire.

Although there was a theory that the counter-earth was introduced in order to bring the number of the moving bodies up to ten, the perfect number according to the Pythagoreans, it is clear from a passage of Aristotle that this was not the real reason. Aristotle says, namely, that the eclipses of the moon were considered to be due sometimes to the interposition of the earth, sometimes to the interposition of the counter-earth. Evidently therefore the purpose of the counter-earth was to explain the frequency with which eclipses of the moon occur.

The Pythagoreans held that the earth, revolving, like one of the stars, about the central fire, makes night and day according to its position relatively to the sun; it is therefore day in that region which is lit up by the sun and night in the cone formed by the earth's shadow. As the same hemisphere is always turned outwards, it follows that the earth completes one revolution about the central fire in a day and a night or in about twenty-four hours. This would account for the apparent diurnal rotation of the heavens from east to west; but for parallax (of which, if we may believe Aristotle, the Pythagoreans made light), it would be equivalent to the rotation of the earth on its own axis once in twenty-four hours. This would make the revolution of the sphere of the fixed stars unnecessary. Yet the Pythagoreans certainly gave some motion to the latter sphere. What it was remains a puzzle. It cannot have been the precession of the equinoxes, for that was first discovered by Hipparchus (second century B.C.). Perhaps there was a real incompatibility between the two revolutions which was unnoticed by the authors of the system.

ŒNOPIDES OF CHIOS.

Œnopides of Chios (a little younger than Anaxagoras) is credited with two discoveries. The first, which was important, was that of the obliquity of the zodiac circle or the ecliptic; the second was that of a Great Year, which Œnopides put at fifty-nine years. He also (so we are told) found the length of the year to be $365\frac{22}{59}$ days. He seems to have obtained this figure by a sort of circular argument. Starting first with 365 days as the length of a year and $29\frac{1}{2}$ days as the length of the lunar month, approximate figures known before his time, he had to find the least integral number of complete years containing an exact number of lunar months; this is clearly fifty-nine years, which contain twice 365 or 730 lunar months. Œnopides seems by his knowledge of the calendar to have determined the number of days in 730 lunar months to be 21,557, and this number divided by fifty-nine, the number of years, gives $365\frac{22}{59}$ as the number of days in the year.

PLATO.

We come now to Plato (427-347 B.C.). In the astronomy of Plato, as we find it in the Dialogues, there is so large an admixture of myth and poetry that it is impossible to be sure what his real views were on certain points of detail. In the *Phædo* we have certain statements about the earth to the effect that it is of very large dimensions, the apparent hollow (according to Plato) in which we live being a very small portion of the whole, and that it is in the middle of the heaven, in equilibrium, without any support, by virtue of the uniformity in the substance of the heaven. In the *Republic* we have a glimpse of a more complete astronomical system. The outermost revolution is that of the sphere of the fixed stars, which carries round with it the whole universe including the sun, moon and planets; the latter seven bodies, while they are so carried round by the general rotation, have slower revolutions of their own in addition, one inside the other, these revolutions being at different speeds but all in the opposite sense to the general rotation of the universe. The quickest rotation is that of the fixed stars and the universe, which takes place once in about twenty-four hours. The slower speeds of the sun, moon and planets are not absolute but relative to the sphere of the fixed stars regarded as stationary. The earth in the centre is unmoved; the successive revolutions about it and within the sphere of the fixed stars are (reckoning from the earth outwards) those of the moon, the sun, Venus, Mercury, Mars, Jupiter, Saturn; the speed of the moon is the quickest, that of the sun the next quickest, while Venus and Mercury travel with the sun and have the same speed, taking about a year to describe their orbits; after these in speed comes Mars, then Jupiter and, last and slowest of all, Saturn. There is nothing said in the *Republic* about the seven bodies revolving in a circle different from and inclined to the equator of the sphere of the fixed stars; that is, the obliquity of the ecliptic does not appear; hence the standpoint of the whole system is that of Pythagoras as distinct from that of the Pythagoreans.

Plato's astronomical system is, however, most fully developed in the *Timæus*. While other details remain substantially the same, the zodiac circle in which the sun, moon and planets revolve is distinguished from the equator of the sphere of the fixed stars. The latter is called the circle of the Same, the former that of the Other, and we are told (quite correctly) that, since the revolution of the universe in the circle of the Same carries all the other revolutions with it, the effect on each of the seven bodies is to turn their actual motions in space into spirals. There is a difficulty in interpreting a phrase in Plato's description which says that Venus and Mercury, though moving in a circle having equal speed with the sun, "have the contrary tendency to it". Literally this would seem to mean that Venus and Mercury describe their circles the opposite way to the sun, but this is so contradicted by observation that Plato could hardly have maintained it; hence the words have been thought to convey a vague reference to the apparent irregularities in the motion of Venus and Mercury, their standings-still and retrogradations.

But the most disputed point in the system is the part assigned in it to the earth. An expression is used with regard to its relation to the axis of the heavenly sphere which might mean either (1) that it is wrapped or globed about that axis but without motion, or (2) that it revolves round the axis. If the word means *revolving* about the axis of the sphere, the revolution would be either (*a*) rotation about its own axis supposed to be identical with that of the sphere, or (*b*) revolution about the axis of the heavenly sphere in the same way that the sun, moon and planets revolve about an axis obliquely inclined to that axis. But (*a*) if the earth rotated about its own axis, this would make unnecessary the rotation of the sphere of the fixed stars once in twenty-four hours, which, however, is expressly included as part of the system. The hypothesis (*b*) would make the system similar to the Pythagorean except that the earth would revolve about the axis of the heavenly sphere instead of round the central fire. The supporters of this hypothesis cite two passages of Plutarch to the effect that Plato was said in his old age to have repented of having given the earth the middle place in the universe instead of placing it elsewhere and giving the middle and chiefest place to some worthier occupant. It is a sufficient answer to this argument that, if Plato really meant in the passage of the *Timæus* to say that the earth revolves about the axis of the heavenly sphere, he had nothing to repent of. We must therefore, for our part, conclude that in his written Dialogues Plato regarded the earth as *at rest* in the centre of the universe.

We have it on good authority that Plato set it as a problem to all earnest students "to find what are the uniform and ordered movements by the assumption of which the apparent movements of the planets can be accounted for". The same authority adds that Eudoxus was the first to formulate a theory with this object; and Heraclides of Pontus followed with an entirely new hypothesis. Both were pupils of Plato and, in so far as the statement of his problem was a stimulus to these speculations, he rendered an important service to the science of astronomy.

EUDOXUS, CALLIPPUS, ARISTOTLE.

Eudoxus of Cnidos (about 408-355 B.C.) was one of the very greatest of the Greek mathematicians. He was the discoverer and elaborator of the great theory of proportion applicable to all magnitudes whether commensurable or incommensurable which is given in Euclid's Book V. He was also the originator of the powerful *method of exhaustion* used by all later Greek geometers for the purpose of finding the areas of curves and the volumes of pyramids, cones, spheres and other curved surfaces. It is not therefore surprising that he should have invented a remarkable geometrical hypothesis for explaining the irregular movements of the planets. The problem was to find the necessary number of circular motions which by their combination would produce the motions of the planets as actually observed, and in particular the variations in their apparent speeds, their stations and retrogradations and their movements in latitude. This Eudoxus endeavoured to do by combining the motions of several concentric spheres, one inside the other, and revolving about different axes, each sphere revolving on its own account but also being carried round bodily by the revolution of the next sphere encircling it. We are dependent on passages from Aristotle and Simplicius for our knowledge of Eudoxus's system, which he had set out in a work *On Speeds*, now lost. Eudoxus assumed three revolving spheres for producing the apparent motions of the sun and moon respectively, and four for that of each of the planets. In his hypothesis for the sun he seems deliberately to have ignored the discovery made by Meton and Euctemon some sixty or seventy years before that the sun does not take the same time to describe the four quadrants of its orbit between the equinoctial and solstitial points.

It should be observed that the whole hypothesis of the concentric spheres is pure geometry, and there is no mechanics in it. We will shortly describe the arrangement of the four spheres which by their revolution produced the motion of a planet. The first and outermost sphere produced the daily rotation in twenty-four hours; the second sphere revolved about an axis perpendicular to the plane of the zodiac or ecliptic, thereby producing the motion along the zodiac "in the respective periods in which the planets appear to describe the zodiac circle," i.e. in the case of the superior planets, the sidereal periods of revolution, and in the case of Mercury and Venus (on a geocentric system) one year. The third sphere had its poles at two opposite points on the zodiac circle, the poles being carried round in the motion of the second sphere; the revolution of the third sphere about the axis connecting the two poles was again uniform and took place in a period equal to the synodic period of the planet, or the time elapsing between two successive oppositions or conjunctions with the sun.

The poles of the third sphere were different for all the planets, except that for Mercury and Venus they were the same. On the surface of the third sphere the poles of the fourth sphere were fixed, and its axis of revolution was inclined to that of the former at an angle constant for each planet but different for the different planets. The planet was fixed at a point on the equator of the fourth sphere. The third and fourth spheres together cause the planet's movement in latitude. Simplicius explains clearly the effect of these two rotations. If, he says, the planet had been on the third sphere (by itself), it would actually have arrived at the poles of the zodiac circle; but, as things are, the fourth sphere, which turns about the poles of the inclined circle carrying the planet and rotates in the opposite sense to the third, i.e. from east to west, but in the same period, will prevent any considerable divergence on the part of the planet from the zodiac circle, and will cause the planet to describe about this same zodiac circle the curve called by Eudoxus the *hippopede* (horse-fetter), so that the breadth of this curve will be the maximum amount of the apparent deviation of the planet in latitude. The curve in question is an elongated figure-of-eight lying along and bisected by the zodiac circle. The motion then round this figure-of-eight combined with the motion in the zodiac circle produces the acceleration and retardation of the motion of the planet, causing the stations and retrogradations. Mathematicians will appreciate the wonderful ingenuity and beauty of the construction.

Eudoxus spent sixteen months in Egypt about 381-380 B.C., and, while there, he assimilated the astronomical knowledge of the priests of Heliopolis and himself made observations. The Observatory between Heliopolis and Cercesura used by him was still pointed out in Augustus's time; he also had one built at Cnidos. He wrote two books entitled respectively the *Mirror* and the *Phænomena;* the poem of Aratus was, so far as verses 19-732 are concerned, drawn from the *Phænomena* of Eudoxus. He is also credited with the invention of the *arachne* (spider's web) which, however, is alternatively attributed to Apollonius, and which seems to have been a sun-clock of some kind.

Eudoxus's system of concentric spheres was improved upon by Callippus (about 370-300 B.C.), who added two more spheres for the sun and the moon, and one more in the case of each of the three nearer planets, Mercury, Venus and Mars. The two additional spheres in the case of the sun were introduced in order to account for the unequal motion of the sun in longitude; and the purpose in the case of the moon was presumably similar.

Callippus made the length of the seasons, beginning with the vernal equinox, ninety-four, ninety-two, eighty-nine and ninety days respectively, figures much more accurate than those given by Euctemon a hundred years earlier, which were ninety-three, ninety, ninety and ninety-two days respectively.

With Callippus as well as Eudoxus the system of concentric spheres was purely geometrical. Aristotle (384-322 B.C.) thought it necessary to alter it in a mechanical sense; he made the spheres into spherical shells actually in contact with one another, and this made it almost necessary, instead of having independent sets of spheres, one set for each planet, to make all the sets part of one continuous system of spheres. For this purpose he assumed sets of *reacting* spheres between successive sets of the original spheres. E.g. Saturn being carried by a set of four spheres, he had three reacting spheres to neutralise the last three, in order to restore the outermost sphere to act as the first of the four spheres producing the motion of the next lower planet, Jupiter, and so on. The change was hardly an improvement.

Aristotle's other ideas in astronomy do not require detailed notice, except his views about the earth. Although he held firmly to the old belief that the earth is in the centre and remains motionless, he was clear that its shape (like that of the stars and the universe) is spherical, and he had arrived at views about its size sounder than those of Plato. In support of the spherical shape of the earth he used some good arguments based on observation. (1) In partial eclipses of the moon the line separating the dark and bright portions is always circular—unlike the line of demarcation in the phases of the moon which may be straight. (2) Certain stars seen above the horizon in Egypt and in Cyprus are not visible further north, and, on the other hand, certain stars set there which in more northern latitudes remain always above the horizon. As there is so perceptible a change of horizon between places so near to each other, it follows not only that the earth is spherical but also that it is not a very large sphere. Aristotle adds that people are not improbably right when they say that the region about the Pillars of Heracles is joined on to India, one sea connecting them. He quotes a result arrived at by the mathematicians of his time, that the circumference of the earth is 400,000 stades. He is clear that the earth is much smaller than some of the stars, but that the moon is smaller than the earth.

The systems of concentric spheres were not destined to hold their ground for long. In these systems the sun, moon and planets were of necessity always at the same distances from the earth respectively. But it was soon recognised that they did not "save the phenomena," since it was seen that the planets appeared to be at one time nearer and at another time further off. Autolycus of Pitane (who flourished about 310 B.C.) knew this and is said to have tried to explain it; indeed it can hardly have been unknown even to the authors of the concentric theory themselves, for Polemarchus of Cyzicus, almost contemporary with Eudoxus, is said to have been aware of it but to have minimised the difficulty because he preferred the hypothesis of the concentric spheres to others.

Development along the lines of Eudoxus's theory being thus blocked, the alternative was open of seeing whether any modification of the Pythagorean system would give better results. We actually have evidence of revisions of the Pythagorean theory. The first step was to get rid of the counter-earth, and some Pythagoreans did this by identifying the counter-earth with the moon. We hear too of a Pythagorean system in which the central fire was not outside the earth but in the centre of the earth itself. The descriptions of this system rather indicate that in it the earth was supposed to be at rest, without any rotation, in the centre of the universe. This was practically a return to the standpoint of Pythagoras himself. But it is clear that, if the system of Philolaus (or Hicetas) be taken and the central fire be transferred to the centre of the earth (the counter-earth being also eliminated), and if the movements of the earth, sun, moon and planets round the centre be retained without any modification save that which is mathematically involved by the transfer of the central fire to the centre of the earth, the daily revolution of the earth about the central fire is necessarily transformed into a rotation of the earth about its own axis in about twenty-four hours.

HERACLIDES OF PONTUS.

All authorities agree that the theory of the daily rotation of the earth about its own axis was put forward by Heraclides of Pontus (about 388-315 B.C.), a pupil of Plato; with him in some accounts is associated the name of one Ecphantus, a Pythagorean. We are told that Ecphantus asserted "that the earth, being in the centre of the universe, moves about its own centre in an eastward direction," and that "Heraclides of Pontus and Ecphantus the Pythagorean make the earth move, not in the sense of translation, but by way of turning as on an axle, like a wheel, from west to east, about its own centre".

Heraclides was born at Heraclea in Pontus. He went to Athens not later than 364 B.C., and there met Speusippus, who introduced him into the school of Plato. On the death of Speusippus (then at the head of the school) in 338, Xenocrates was elected to succeed him; at this election Heraclides was also a candidate and was only defeated by a few votes. He was the author of dialogues, brilliant and original, on all sorts of subjects, which were much read and imitated at Rome, e.g. by Varro and Cicero. Two of them "On Nature" and "On the Heavens" may have dealt with astronomy.

In his view that the earth rotates about its own axis Heraclides is associated with Aristarchus of Samos; thus Simplicius says: "There have been some, like Heraclides of Pontus and Aristarchus, who supposed that the phenomena can be saved if the heaven and the stars are at rest while the earth moves about the poles of the equinoctial circle from the west to the east, completing one revolution each day, approximately; the 'approximately' is added because of the daily motion of the sun to the extent of one degree".

Heraclides made another important advance towards the Copernican hypothesis. He discovered the fact that Venus and Mercury revolve about the sun as centre. So much is certain; but a further question naturally arises. Having made Venus and Mercury revolve round the sun like satellites, did Heraclides proceed to draw the same inference with regard to the other, the superior, planets? The question is interesting because, had it been laid down that all the five planets alike revolve round the sun, the combination of this hypothesis with Heraclides's assumption that the earth rotates about its own axis in twenty-four hours would have amounted to an anticipation of the system of Tycho Brahe, but with the improvement of the substitution of the daily rotation of the earth for the daily revolution of the whole system about the earth supposed at rest. Schiaparelli dealt with the question in two papers entitled *I precursori di Copernico nell' antichità* (1873), and *Origine del sistema planetario eliocentrico presso i Greci* (1898). Schiaparelli tried to show that Heraclides did arrive at the conclusion that the superior planets as well as Mercury and Venus revolve round the sun; but most persons will probably agree that his argument is not convincing. The difficulties seem too great. The circles described by Mercury and Venus about the sun are relatively small circles and are entirely on one side of the earth. But when the possibility of, say, Mars revolving about the sun came to be considered, it would be at once obvious that the precise hypothesis adopted for Mercury and Venus would not apply. It would be seen that Mars is brightest when it occupies a position in the zodiac *opposite* to the sun; it must therefore be nearest to the earth at that time. Consequently the circle described by Mars, instead of being on one side of the earth, must comprehend the earth which is inside it. Whereas therefore the circles described by Mercury and Venus were what the Greeks called *epicycles* about a material centre, the sun (itself moving in a circle round the earth), what was wanted in the case of Mars (if the circle described by Mars was to have the sun for centre) was what the Greeks called an *eccentric* circle, with a centre which itself moves in a circle about the earth, and with a radius greater than that of the sun's orbit. Though the same motion could have been produced by an *epicycle*, the epicycle would have had to have a mathematical point (not the material sun) as centre. But the idea of using non-material points as centres for epicycles was probably first thought of, at a later stage, by some of the great mathematicians such as Apollonius of Perga (about 265-190 B.C.).

Not only does Schiaparelli maintain that the complete (but improved) Tychonic hypothesis was put forward by Heraclides or at least in Heraclides's time; he goes further and makes a still greater claim on behalf of Heraclides, namely, that it was he, and not Aristarchus of

Samos, who first stated as a possibility the Copernican hypothesis. Now it was much to discover, as Heraclides did, that the earth rotates about its own axis and that Mercury and Venus revolve round the sun like satellites; and it seems *a priori* incredible that one man should not only have reached, and improved upon, the hypothesis of Tycho Brahe but should *also* have suggested the Copernican hypothesis. It is therefore necessary to examine briefly the evidence on which Schiaparelli relied. His argument rests entirely upon one passage, a sentence forming part of a quotation from a summary by Geminus of the *Meteorologica* of Posidonius, which Simplicius copied from Alexander Aphrodisiensis and embodied in his commentary on the *Physics* of Aristotle. The sentence in question, according to the reading of the MSS., is as follows: "Hence we actually find a certain person, Heraclides of Pontus, coming forward and saying that, even on the assumption that the earth moves in a certain way, while the sun is in a certain way at rest, the apparent irregularity with reference to the sun can be saved". (The preceding sentence is about possible answers to the question, why do the sun, the moon and the planets appear to move irregularly? and says, "we may answer that, if we assume that their orbits are eccentric circles or that the stars describe an epicycle, their apparent irregularity will be saved, and it will be necessary to go further and examine in how many different ways it is possible for these phenomena to be brought about".)

Now it is impossible that Geminus himself can have spoken of an astronomer of the distinction of Heraclides as "*a certain* Heraclides of Pontus". Consequently there have been different attempts made to emend the reading of the MSS. All the emendations proposed are open to serious objections, and we are thrown back on the reading of the MSS. Now it "leaps to the eyes" that, if the name of Heraclides of Pontus is left out, everything is in order. "This is why one astronomer has actually suggested that, by assuming the earth to move in a certain way, and the sun to be in a certain way at rest, the apparent irregularity with reference to the sun will be saved." This seems to be the solution of the puzzle suggested by the ordinary principles of textual criticism, and is so simple and natural that it will surely carry conviction to the minds of unbiassed persons. Geminus, in fact, mentioned no name but meant Aristarchus of Samos, and some scholiast, remembering that Heraclides had given a certain motion to the earth (namely, rotation about its axis), immediately thought of Heraclides and inserted his name in the margin, from which it afterwards crept into the text.

It is only necessary to add that Archimedes is not likely to have been wrong when he attributed the first suggestion of the Copernican hypothesis to Aristarchus of Samos in express terms; and this is confirmed by another positive statement by Aëtius, already quoted, that "Heraclides of Pontus and Ecphantus the Pythagorean made the earth move, *not* in the sense of translation, but with a movement of rotation".

PART II.

ARISTARCHUS OF SAMOS.

WE are told that Aristarchus of Samos was a pupil of Strato of Lampsacus, a natural philosopher of originality, who succeeded Theophrastus as head of the Peripatetic school in 288 or 287 B.C., and held that position for eighteen years. Two other facts enable us to fix Aristarchus's date approximately. In 281-280 he made an observation of the summer solstice; and the book in which he formulated his heliocentric hypothesis was published before the date of Archimedes's *Psammites* or *Sandreckoner*, a work written before 216 B.C. Aristarchus therefore probably lived *circa* 310-230 B.C., that is, he came about seventy-five years later than Heraclides and was older than Archimedes by about twenty-five years.

Aristarchus was called "the mathematician," no doubt in order to distinguish him from the many other persons of the same name; Vitruvius includes him among the few great men who possessed an equally profound knowledge of all branches of science, geometry, astronomy, music, etc. "Men of this type are rare, men such as were in times past Aristarchus of Samos, Philolaus and Archytas of Tarentum, Apollonius of Perga, Eratosthenes of Cyrene, Archimedes and Scopinas of Syracuse, who left to posterity many mechanical and gnomonic appliances which they invented and explained on mathematical and natural principles." That Aristarchus was a very capable geometer is proved by his extant book, *On the sizes and distances of the sun and moon*, presently to be described. In the mechanical line he is credited with the invention of an improved sun-dial, the so-called *scaphe*, which had not a plane but a concave hemispherical surface, with a pointer erected vertically in the middle, throwing shadows and so enabling the direction and height of the sun to be read off by means of lines marked on the surface of the hemisphere. He also wrote on vision, light, and colours. His views on the latter subjects were no doubt largely influenced by the teaching of Strato. Strato held that colours were emanations from bodies, material molecules as it were, which imparted to the intervening air the same colour as that possessed by the body. Aristarchus said that colours are "shapes or forms stamping the air with impressions like themselves as it were," that "colours in darkness have no colouring," and that "light is the colour impinging on a substratum".

THE HELIOCENTRIC HYPOTHESIS.

There is no doubt whatever that Aristarchus put forward the heliocentric hypothesis. Ancient testimony is unanimous on the point, and the first witness is Archimedes who was a younger contemporary of Aristarchus, so that there is no possibility of a mistake. Copernicus himself admitted that the theory was attributed to Aristarchus, though this does not seem to be generally known. Copernicus refers in two passages of his work, *De revolutionibus caelestibus*, to the opinions of the ancients about the motion of the earth. In the dedicatory letter to Pope Paul III he mentions that he first learnt from Cicero that one Nicetas (i.e. Hicetas) had attributed motion to the earth, and that he afterwards read in

Plutarch that certain others held that opinion; he then quotes the *Placita philosophorum* according to which "Philolaus the Pythagorean asserted that the earth moved round the fire in an oblique circle in the same way as the sun and moon". In Book I. c. 5 of his work Copernicus alludes to the views of Heraclides, Ecphantus, and Hicetas, who made the earth rotate about its own axis, and then goes on to say that it would not be very surprising if any one should attribute to the earth another motion besides rotation, namely, revolution in an orbit in space: "atque etiam (terram) pluribus motibus vagantem et unam ex astris Philolaus Pythagoricus sensisse fertur, Mathematicus non vulgaris". Here, however, there is no question of the earth revolving round the sun, and there is no mention of Aristarchus. But Copernicus did mention the theory of Aristarchus in a passage which he afterwards suppressed: "Credibile est hisce similibusque causis Philolaum mobilitatem terrae sensisse, quod etiam nonnulli Aristarchum Samium ferunt in eadem fuisse sententia".

It is desirable to quote the whole passage of Archimedes in which the allusion to Aristarchus's heliocentric hypothesis occurs, in order to show the whole context.

"You are aware ['you' being King Gelon] that 'universe' is the name given by most astronomers to the sphere the centre of which is the centre of the earth, while its radius is equal to the straight line between the centre of the sun and the centre of the earth. This is the common account as you have heard from astronomers. But Aristarchus brought out *a book consisting of certain hypotheses*, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the 'universe' just mentioned. His hypotheses are that *the fixed stars and the sun remain unmoved, that the earth revolves about the sun in the circum-*

ference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface."

The heliocentric hypothesis is here stated in language which leaves no room for doubt about its meaning. The sun, like the fixed stars, remains unmoved and forms the centre of a circular orbit in which the earth moves round it; the sphere of the fixed stars has its centre at the centre of the sun.

We have further evidence in a passage of Plutarch's tract, *On the face in the moon's orb*: "Only do not, my dear fellow, enter an action for impiety against me in the style of Cleanthes, who thought it was the duty of Greeks to indict Aristarchus on the charge of impiety for putting in motion the Hearth of the Universe, this being the effect of his attempt to save the phenomena by supposing the heaven to remain at rest and the earth to revolve in an oblique circle, while it rotates, at the same time, about its own axis".

Here we have the additional detail that Aristarchus followed Heraclides in attributing to the earth the daily rotation about its axis; Archimedes does not state this in so many words, but it is clearly involved by his remark that Aristarchus supposed the fixed stars as well as the sun to remain unmoved in space. A tract "Against Aristarchus" is mentioned by Diogenes Laertius among Cleanthes's works; and it was evidently published during Aristarchus's lifetime (Cleanthes died about 232 B.C.).

We learn from another passage of Plutarch that the hypothesis of Aristarchus was adopted, about a century later, by Seleucus, of Seleucia on the Tigris, a Chaldæan or Babylonian, who also wrote on the subject of the tides about 150 B.C. The passage is interesting because it also alludes to the doubt about Plato's final views. "Did Plato put the earth in motion as he did the sun, the moon and the five planets which he called the 'instruments of time' on account of their turnings, and was it necessary to conceive that the earth 'which is globed about the axis stretched from pole to pole through the whole universe' was not represented as being (merely) held together and at rest but as turning and revolving, as Aristarchus and Seleucus afterwards maintained that it did, the former of whom stated this as only a hypothesis, the latter as a definite opinion?"

No one after Seleucus is mentioned by name as having accepted the doctrine of Aristarchus and, if other Greek astronomers refer to it, they do so only to denounce it. Hipparchus, himself a contemporary of Seleucus, definitely reverted to the geocentric system, and it was doubtless his authority which sealed the fate of the heliocentric hypothesis for so many centuries.

The reasons which weighed with Hipparchus were presumably the facts that the system in which the earth revolved in a circle of which the sun was the exact *centre* failed to "save the phenomena," and in particular to account for the variations of distance and the irregularities of the motions, which became more and more patent as methods of observation improved; that, on the other hand, the theory of epicycles did suffice to represent the phenomena with considerable accuracy; and that the latter theory could be reconciled with the immobility of the earth.

ON THE APPARENT DIAMETER OF THE SUN.

Archimedes tells us in the same treatise that "Aristarchus discovered that the sun's apparent size is about

$\frac{1}{720}$th part of the zodiac circle"; that is to say, he observed that the angle subtended at the earth by the diameter of the sun is about half a degree.

ON THE SIZES AND DISTANCES OF THE SUN AND MOON.

Archimedes also says that, whereas the ratio of the diameter of the sun to that of the moon had been estimated by Eudoxus at 9 : 1 and by his own father Phidias at 12 : 1, Aristarchus made the ratio greater than 18 : 1 but less than 20 : 1. Fortunately we possess in Greek the short treatise in which Aristarchus proved these conclusions; on the other matter of the apparent diameter of the sun Archimedes's statement is our only evidence.

It is noteworthy that in Aristarchus's extant treatise *On the sizes and distances of the sun and moon* there is no hint of the heliocentric hypothesis, while the apparent diameter of the sun is there assumed to be, not $\frac{1}{2}^{\circ}$, but the very inaccurate figure of 2°. Both circumstances are explained if we assume that the treatise was an early work written before the hypotheses described by Archimedes were put forward. In the treatise Aristarchus finds the ratio of the diameter of the sun to the diameter of the earth to lie between 19 : 3 and 43 : 6; this would make the volume of the sun about 300 times that of the earth, and it may be that the great size of the sun in comparison with the earth, as thus brought out, was one of the considerations which led Aristarchus to place the sun rather than the earth in the centre of the universe, since it might even at that day seem absurd to make the body which was so much larger revolve about the smaller.

There is no reason to doubt that in his heliocentric system Aristarchus retained the moon as a satellite of the earth revolving round it as centre; hence even in his system there was one *epicycle.*

The treatise *On sizes and distances* being the only work of Aristarchus which has survived, it will be fitting to give here a description of its contents and special features.

The style of Aristarchus is thoroughly classical as befits an able geometer intermediate in date between Euclid and Archimedes, and his demonstrations are worked out with the same rigour as those of his predecessor and successor. The propositions of Euclid's *Elements* are, of course, taken for granted, but other things are tacitly assumed which go beyond what we find in Euclid. Thus the transformations of ratios defined in Euclid, Book V, and denoted by the terms *inversely*, *alternately*, *componendo*, *convertendo*, etc., are regularly used in dealing with *unequal* ratios, whereas in Euclid they are only used in proportions, i.e. cases of *equality* of ratios. But the propositions of Aristarchus are also of particular mathematical interest because the ratios of the sizes and distances which have to be calculated are really trigonometrical ratios, sines, cosines, etc., although at the time of Aristarchus trigonometry had not been invented, and no reasonably close approximation to the value of π, the ratio of the circumference of any circle to its diameter, had been made (it was Archimedes who first obtained the approximation $\frac{22}{7}$). Exact calculation of the trigonometrical ratios being therefore impossible for Aristarchus, he set himself to find upper and lower limits for them, and he succeeded in locating those which emerge in his propositions within tolerably narrow limits, though not always the narrowest within which it would have been possible, even for him, to confine them. In this species of approximation to trigonometry he tacitly assumes propositions comparing the ratio between a greater and a lesser *angle* in a figure with the ratio between two *straight lines*, propositions which are formally proved by Ptolemy at the beginning of his *Syntaxis*. Here again we have proof that textbooks containing such propositions existed before Aristarchus's time, and probably much earlier, although they have not survived.

Aristarchus necessarily begins by laying down, as the basis for his treatise, certain assumptions. They are six in number, and he refers to them as *hypotheses*. We cannot do better than quote them in full, along with the sentences immediately following, in which he states the main results to be established in the treatise:—

[HYPOTHESES.]

1. *That the moon receives its light from the sun.*

2. *That the earth is in the relation of a point and centre to the sphere in which the moon moves.*

3. *That, when the moon appears to us halved, the great circle which divides the dark and the bright portions of the moon is in the direction of our eye.*

4. *That, when the moon appears to us halved, its distance from the sun is then less than a quadrant by one-thirtieth of a quadrant.*

5. *That the breadth of the (earth's) shadow is (that) of two moons.*

6. *That the moon subtends one-fifteenth part of a sign of the zodiac.*

We are now in a position to prove the following propositions:—

1. *The distance of the sun from the earth is greater than eighteen times, but less than twenty times, the distance of the moon (from the earth)*; this follows from the hypothesis about the halved moon.

2. *The diameter of the sun has the same ratio (as aforesaid) to the diameter of the moon.*

3. *The diameter of the sun has to the diameter of the earth a ratio greater than that which 19 has to 3, but less than that which 43 has to 6*; this follows from the ratio thus discovered between the distances, the hypothesis about the shadow, and the hypothesis that the moon subtends one-fifteenth part of a sign of the zodiac.

The first assumption is Anaxagoras's discovery. The second assumption is no doubt an exaggeration; but it is made in order to avoid having to allow for the fact that the phenomena as seen by an observer on the surface of the earth are slightly different from what would be seen if the observer's eye were at the centre of the earth. Aristarchus, that is, takes the earth to be like a point in order to avoid the complication of parallax.

The meaning of the third hypothesis is that the plane of the great circle in question passes through the point where the eye of the observer is situated; that is to say, we see the circle *end on*, as it were, and it looks like a straight line.

Hypothesis 4. If S be the sun, M the moon and E the earth, the triangle SME is, at the moment when the moon appears to us halved, right-angled at M; and the hypothesis states that the angle at E in this triangle is 87°, or, in other words, the angle MSE, that is, the angle subtended at the sun by the line joining M to E, is 3°. These estimates are decidedly inaccurate, for the true value of the angle MES is 89° 50′, and that of the angle MSE is therefore 10′. There is nothing to show how Aristarchus came to estimate the angle MSE at 3°, and none of his successors seem to have made any direct estimate of the size of the angle.

The assumption in Hypothesis 5 was improved upon later. Hipparchus made the ratio of the diameter of the circle of the earth's shadow to the diameter of the moon to be, not 2, but 2½ at the moon's mean distance at the conjunctions; Ptolemy made it, at the moon's greatest distance, to be inappreciably less than $2\frac{5}{8}$.

The sixth hypothesis states that the diameter of the moon subtends at our eye an angle which is $\frac{1}{15}$th of 30°, i.e. 2°, whereas Archimedes, as we have seen, tells us that Aristarchus found the angle subtended by the diameter of the sun to be $\frac{1}{2}$° (Archimedes in the same tract describes a rough instrument by means of which he himself found that the diameter of the sun subtended an angle less than $\frac{1}{164}$th, but greater than $\frac{1}{200}$th of a right angle). Even the Babylonians had, many centuries before, arrived at 1° as the apparent angular diameter of the sun. It is not clear why Aristarchus took a value so inaccurate as 2°. It has been suggested that he merely intended to give a specimen of the calculations which would have to be made on the basis of more exact experimental observations, and to show that, for the solution of the problem, one of the data could be chosen almost arbitrarily, by which proceeding he secured himself against certain objections which might have been raised. Perhaps this is too ingenious, and it may be that, in view of the difficulty of working out the geometry if the two angles in question are very small, he took 3° and 2° as being the smallest with which he could conveniently deal. Certain it is that the *method* of Aristarchus is perfectly correct and, if he could have substituted the true values (as we know them to-day) for the inaccurate values which he assumes, and could have carried far enough his geometrical substitute for trigonometry, he would have obtained close limits for the true sizes and distances.

The book contains eighteen propositions. Prop. 1 proves that we can draw one cylinder to touch two equal spheres, and one cone to touch two unequal spheres, the planes of the circles of contact being at right angles to the axis of the cylinder or cone. Next (Prop. 2) it is shown that, if a lesser sphere be illuminated by a greater, the illuminated portion of the former will be greater than a hemisphere. Prop. 3 proves that the circle in the moon which divides the dark and the bright portions (we will in future, for short, call this "the dividing circle") is least when the cone which touches the sun and the moon has its vertex at our eye. In Prop. 4 it is shown that the dividing circle is not perceptibly different from a great circle in the moon. If CD is a diameter of the dividing circle, EF the parallel diameter of the parallel great circle in the moon, O the centre of the moon, A the observer's eye, FDG the great circle in the moon the plane of which passes through A, and G the point where OA meets the latter great circle, Aristarchus takes an arc of the great circle GH on one side of G, and another GK on the other side of G, such that GH = GK = $\frac{1}{2}$ (the arc FD), and proves that the angle subtended at A by the arc HK is less than $\frac{1}{45}$°; consequently, he says, the arc would be imperceptible at A even in that position, and *a fortiori* the arc FD (which is nearly in a straight line with the tangent AD) is quite imperceptible to the observer at A. Hence (Prop. 5), when the moon appears to us halved, we can take the plane of the great circle in the moon which is parallel to the dividing circle as passing through our eye. (It is tacitly assumed in Props. 3, 4, and throughout, that the diameters of the sun and moon respectively subtend the same angle at our eye.) The proof of Prop. 4 assumes as known the equivalent of the proposition in trigonometry that, if each of the angles α, β is not greater than a right angle, and $\alpha > \beta$, then

$$\frac{\tan \alpha}{\tan \beta} > \frac{\alpha}{\beta} > \frac{\sin \alpha}{\sin \beta}$$

Prop. 6 proves that the moon's orbit is "lower" (i.e. smaller) than that of the sun, and that, when the moon appears to us halved, it is distant less than a quadrant from the sun. Prop. 7 is the main proposition in the treatise. It proves that, on the assumptions made, the distance of the sun from the earth is greater than eighteen times, but less than twenty times, the distance of the moon from the earth. The proof is simple and elegant and should delight any mathematician; its two parts depend respectively on the geometrical equivalents of the two inequalities stated in the formula quoted above, namely,

$$\frac{\tan \alpha}{\tan \beta} > \frac{\alpha}{\beta} > \frac{\sin \alpha}{\sin \beta},$$

where α, β are angles not greater than a right angle and $\alpha > \beta$. Aristarchus also, in this proposition, cites $\frac{7}{5}$ as an approximation by defect to the value of $\sqrt{2}$, an approximation found by the Pythagoreans and quoted by Plato. The trigonometrical equivalent of the result obtained in Prop. 7 is

$$\frac{1}{18} > \sin 3^\circ > \frac{1}{20}.$$

Prop. 8 states that, when the sun is totally eclipsed, the sun and moon are comprehended by one and the same cone which has its vertex at our eye. Aristarchus supports this by the arguments (1) that, if the sun overlapped the moon, it would not be totally eclipsed, and (2) that, if the sun fell short (i.e. was more than covered), it would remain totally eclipsed for some time, which it does not (this, he says, is manifest from observation). It is clear from this reasoning that Aristarchus had not observed the phenomenon of an *annular* eclipse of the sun; and it is curious that the first mention of an annular eclipse seems to be that quoted by Simplicius from

Sosigenes (second century, A.D.), the teacher of Alexander Aphrodisiensis.

It follows (Prop. 9) from Prop. 8 that the diameters of the sun and moon are in the same ratio as their distances from the earth respectively, that is to say (Prop. 7) in a ratio greater than 18 : 1 but less than 20 : 1. Hence (Prop. 10) the volume of the sun is more than 5832 times and less than 8000 times that of the moon.

By the usual geometrical substitute for trigonometry Aristarchus proves in Prop. 11 that the diameter of the moon has to the distance between the centre of the moon and our eye a ratio which is less than $\frac{2}{45}$ths but greater than $\frac{1}{30}$th. Since the angle subtended by the moon's diameter at the observer's eye is assumed to be 2°, this proposition is equivalent to the trigonometrical formula

$$\frac{1}{45} > \sin 1^\circ > \frac{1}{60}.$$

Having proved in Prop. 4 that, so far as our perception goes, the dividing circle in the moon is indistinguishable from a great circle, Aristarchus goes behind perception and proves in Prop. 12 that the diameter of the dividing circle is less than the diameter of the moon but greater than $\frac{89}{90}$ths of it. This is again because half the angle subtended by the moon at the eye is assumed to be 1° or $\frac{1}{90}$th of a right angle. The proposition is equivalent to the trigonometrical formula

$$1 > \cos 1^\circ > \frac{89}{90}.$$

We come now to propositions which depend on Hypothesis 5 that "the breadth of the earth's shadow is that of two moons". Prop. 13 is about the diameter of the circular section of the cone formed by the earth's shadow at the place where the moon passes through it in an eclipse, and it is worth while to notice the extreme accuracy with which Aristarchus describes the diameter in question. It is with him "the straight line subtending the portion intercepted within the earth's shadow of the circumference of the circle in which the extremities of the diameter of the circle dividing the dark and the bright portions in the moon move". Aristarchus proves that the length of the straight line in question has to the diameter of the moon a ratio less than 2 but greater than 88 : 45, and has to the diameter of the sun a ratio less than 1 : 9 but greater than 22 : 225. The ratio of the straight line to the diameter of the moon is, in point of fact, $2 \cos^2 1°$ or $2 \sin^2 89°$, and Aristarchus therefore proves the equivalent of

$$2 > 2 \cos^2 1° > \frac{1}{2}\left(\frac{89}{45}\right)^2 \text{ or } \frac{7921}{4050}.$$

He then observes (without explanation) that $\frac{7921}{4050} > \frac{88}{45}$ (an approximation easily obtained by developing $\frac{7921}{4050}$ as a continued fraction $\left(= 1 + \frac{1}{1 +} \frac{1}{21 +} \frac{1}{2}\right)$; his result is therefore equivalent to

$$1 > \cos^2 1° > \frac{44}{45}.$$

The next propositions are the equivalents of more complicated trigonometrical formulæ. Prop. 14 is an auxiliary proposition to Prop. 15. The diameter of the shadow dealt with in Prop. 13 divides into two parts the straight line joining the centre of the earth to the centre of the moon, and Prop. 14 shows that the whole length of this line is more than 675 times the part of it terminating in the centre of the moon. With the aid of Props. 7, 13, and 14 Aristarchus is now able, in Prop.

15, to prove another of his main results, namely, that the diameter of the sun has to the diameter of the earth a ratio greater than 19 : 3 but less than 43 : 6. In the second half of the proof he has to handle quite large numbers. If A be the centre of the sun, B the centre of the earth, and M the vertex of the cone formed by the earth's shadow, he proves that MA : AB is greater than (10125 × 7087) : (9146 × 6750) or 71755875 : 61735500, and then adds, without any word of explanation, that the latter ratio is greater than 43 : 37. Here again it is difficult not to see in 43 : 37 the continued fraction $1 + \frac{1}{6 + \frac{1}{6}}$; and although we cannot suppose that the Greeks could actually develop $\frac{71755875}{61735500}$ or $\frac{21261}{18292}$ as a continued fraction (in *form*), "we have here an important proof of the employment by the ancients of a method of calculation, the theory of which unquestionably belongs to the moderns, but the first applications of which are too simple not to have originated in very remote times" (Paul Tannery).

The remaining propositions contain no more than arithmetical inferences from the foregoing. Prop. 16 is to the effect that the volume of the sun has to the volume of the earth a ratio greater than 6859 : 27 but less than 79507 : 216 (the numbers are the cubes of those in Prop. 15); Prop. 17 proves that the diameter of the earth is to that of the moon in a ratio greater than 108 : 43 but less than 60 : 19 (ratios compounded of those in Props. 9 and 15), and Prop. 18 proves that the volume of the earth is to that of the moon in a ratio greater than 1259712 : 79507 but less than 216000 : 6859.

ARISTARCHUS ON THE YEAR AND "GREAT YEAR".

Aristarchus is said to have increased by $\frac{1}{1623}$rd of a day Callippus's figure of $365\frac{1}{4}$ days as the length of the solar year, and to have given 2484 years as the length of the Great Year or the period after which the sun, the moon and the five planets return to the same position in the heavens. Tannery has shown reason for thinking that 2484 is a wrong reading for 2434 years, and he gives an explanation which seems convincing of the way in which Aristarchus arrived at 2434 years as the length of the Great Year. The Chaldæan period of 223 lunations was well known in Greece. Its length was calculated to be $6585\frac{1}{3}$ days, and in this period the sun was estimated to describe $10\frac{2}{3}°$ of its circle in addition to 18 sidereal revolutions. The Greeks used the period called by them *exeligmus* which was three times the period of 223 lunations and contained a whole number of days, namely, 19756, during which the sun described 32° in addition to 54 sidereal revolutions. It followed that the number of days in the sidereal year was—

$$\frac{19756}{54 + \frac{32}{360}} = \frac{19756}{54 + \frac{4}{45}} = \frac{45 \times 19756}{2434} = \frac{889020}{2434}$$

$$= 365\tfrac{1}{4} + \frac{3}{4868}.$$

Now $\frac{4868}{3} = 1623 - \frac{1}{3}$, and Aristarchus seems to have merely replaced $\frac{3}{4868}$ by the close approximation $\frac{1}{1623}$. The calculation was, however, of no value because the estimate of $10\frac{2}{3}°$ over 18 sidereal revolutions seems to have been an approximation based merely on the difference between $6585\frac{1}{3}$ days and 18 years of $365\frac{1}{4}$ days, i.e. $6574\frac{1}{2}$ days; thus the $10\frac{2}{3}°$ itself probably depended on a solar year of $365\frac{1}{4}$ days, and Aristarchus's evaluation of it as $365\frac{1}{4}\frac{1}{1623}$ was really a sort of circular argument like the similar calculation of the length of the year made by Œnopides of Chios.

LATER IMPROVEMENTS ON ARISTARCHUS'S FIGURES.

It may interest the reader to know how far Aristarchus's estimates of sizes and distances were improved upon by later Greek astronomers. We are not informed how large he conceived the earth to be; but Archimedes tells us that "some have tried to prove that the circumference of the earth is about 300,000 stades and not greater," and it may be presumed that Aristarchus would, like Archimedes, be content with this estimate. It is probable that it was Dicaearchus who (about 300 B.C.) arrived at this value, and that it was obtained by taking 24° ($\frac{1}{15}$th of the whole meridian circle) as the difference of latitude between Syene and Lysimachia (on the same meridian) and 20,000 stades as the actual distance between the two places. Eratosthenes, born a few years after Archimedes, say 284 B.C., is famous for a better measurement of the earth which was based on scientific principles. He found that at noon at the summer solstice the sun threw no shadow at Syene, whereas at the same hour at Alexandria (which he took to be on the same meridian) a vertical stick cast a shadow corresponding to $\frac{1}{50}$th of the meridian circle. Assuming then that the sun's rays at the two places are parallel in direction, and knowing the distance between them to be 5000 stades, he had only to take 50 times 5000 stades to get the circumference of the earth. He seems, for some reason, to have altered 250,000 into 252,000 stades, and this, according to Pliny's account of the kind of stade used, works out to about 24,662 miles, giving for the diameter of the earth a length of 7850 miles, a surprisingly close approximation, however much it owes to happy accidents in the calculation.

Eratosthenes's estimates of the sizes and distances of the sun and moon cannot be restored with certainty in view of the defective state of the texts of our authorities. We are better informed of Hipparchus's results. In the first book of a treatise on sizes and distances Hipparchus based himself on an observation of an eclipse of the sun, probably that of 20th November in the year 129 B.C., which was exactly total in the region about the Hellespont, whereas at Alexandria about $\frac{4}{5}$ths only of the diameter was obscured. From these facts Hipparchus deduced that, if the radius of the earth be the unit, the least distance of the moon contains 71, and the greatest 83 of these units, the mean thus containing 77. But he reverted to the question in the second book and proved "from many considerations" that the mean distance of the moon is $67\frac{1}{3}$ times the radius of the earth, and also that the distance of the sun is 2490 times the radius of the earth. Hipparchus also made the size (meaning thereby the solid content) of the sun to be 1880 times that of the earth, and the size of the earth to be 27 times that of the moon. The cube root of 1880 being about $12\frac{1}{3}$, the diameters of the sun, earth and moon would be in the ratio of the numbers $12\frac{1}{3}$, 1, $\frac{1}{3}$. Hipparchus seems to have accepted Eratosthenes's estimate of 252,000 stades for the circumference of the earth.

It is curious that Posidonius (135-51 B.C.), who was much less of an astronomer, made a much better guess at the distance of the sun from the earth. He made it 500,000,000 stades. As he also estimated the circumference of the earth at 240,000 stades, we may take the diameter of the earth to be, according to Posidonius, about 76,400 stades; consequently, if D be that diameter, Posidonius made the distance of the sun to be equal to 6545D as compared with Hipparchus's 1245D.

Ptolemy does not mention Hipparchus's figures. His own estimate of the sun's distance was 605D, so that Hipparchus was far nearer the truth. But Hipparchus's estimate remained unknown and Ptolemy's held the field for many centuries; even Copernicus only made the distance of the sun 750 times the earth's diameter, and it was not till 1671-3 that a substantial improvement was made; observations of Mars carried out in those years by Richer enabled Cassini to conclude that the sun's parallax was about 9·5″ corresponding to a distance between the sun and the earth of 87,000,000 miles.

Ptolemy made the distance of the moon from the earth to be 29½ times the earth's diameter, and the diameter of the earth to be 3⅖ times that of the moon. He estimated the diameter of the sun at 18⅘ times that of the moon and therefore about 5½ times that of the earth, a figure again much inferior to that given by Hipparchus.

BIBLIOGRAPHY.

On the astronomy of the early Greek philosophers much information is given by Aristotle (especially in the *De Caelo*); for Aristotle was fortunately in the habit of stating the views of earlier thinkers as a preliminary to enunciating his own. Apart from what we learn from Aristotle, we are mainly dependent on the fragmentary accounts of the opinions of philosophers which were collected in the *Doxographi Graeci* of Hermann Diels (Berlin, 1879), to which must be added, for the period before Socrates, *Die Fragmente der Vorsokratiker* by the same editor (2nd edition, with index, 1906-10, 3rd edition, 1912). The doxographic data and the fragments for the period from Thales to Empedocles were translated and explained by Paul Tannery in *Pour l'histoire de la Science Hellène* (Paris, 1887). A History of Astronomy was written by Eudemus of Rhodes, a pupil of Aristotle; this is lost, but a fair number of fragments are preserved by later writers. Of the theory of concentric spheres we have a short account in Aristotle's *Metaphysics*, but a more elaborate and detailed description is contained in Simplicius's commentary on the *De Caelo*; Simplicius quotes largely from Sosigenes the Peripatetic (second century A.D.) who drew upon Eudemus.

There are a number of valuable histories of Greek astronomy. In German we have Schaubach, *Geschichte der griechischen Astronomie bis auf Eratosthenes*, 1809; R. Wolf, *Geschichte der Astronomie*, 1877; and two admirable epitomes (1) by Siegmund Günther in Windelband's *Geschichte der alten Philosophie* (Iwan von Müller's *Handbuch der klassischen Altertumswissenschaft*, Vol. V, Pt. 1), 2nd edition, 1894, and (2) by Friedrich Hultsch in Pauly-Wissowa's *Real-Encyclopädie der classischen Altertumswissenschaft* (Art. Astronomie in Vol. II, 2, 1896).

In French, besides the great work of Delambre, *Histoire de l'astronomie ancienne*, 1817, we have the valuable studies of Paul Tannery in *Recherches sur l'histoire de l'astronomie ancienne*, 1893, and Pierre Duhem, *Le Système du Monde*, Vol. I, 1913.

In English, reference may be made to Sir G. Cornewall Lewis, *An Historical Survey of the Astronomy of the Ancients*, 1863; J. L. E. Dreyer, *History of the Planetary Systems from Thales to Kepler*, Cambridge, 1906; and the historical portion of Sir Thomas Heath's *Aristarchus of Samos, the ancient Copernicus*, Oxford, 1913.

Aristarchus's treatise *On the Sizes and Distances of the Sun and Moon* first appeared in a Latin translation by George Valla in 1488 and 1498, and next in a Latin translation by Commandinus (1572). The *editio princeps* of the Greek text was brought out by John Wallis, Oxford, 1688, and was reprinted in *Johannis Wallis Opera Mathematica*, 1693-1699, Vol. III, in both cases along with Commandinus's translation. In 1810 there appeared an edition by the Comte de Fortia d'Urban, *Histoire d'Aristarque de Samos* . . . including the Greek text and Commandinus's translation but without figures; a French translation by Fortia d'Urban followed in 1823. The treatise was translated into German by A. Nokk in 1854. Finally, Sir Thomas Heath's work above cited contains a new Greek text with English translation and notes.

CHRONOLOGY.

(*Approximate where precise dates are not known.*)

B.C.	
624-547	Thales.
610-546	Anaximander.
585-526	Anaximenes.
572-497	Pythagoras.
born 516 or 514 (possibly 540).	Parmenides.
500-428	Anaxagoras.
494-434	Empedocles.
5th century	Œnopides of Chios. Philolaus.
427-347	Plato.
408-355	Eudoxus.
388-315	Heraclides of Pontus.
384-322	Aristotle.
370-300	Callippus.
310-230	Aristarchus of Samos.
287-212	Archimedes.
284-203	Eratosthenes.
265-190	Apollonius of Perga.
3rd century	Aratus.
fl. 150	Hipparchus.
135-51	Posidonius.
A.D.	
50-125	Plutarch.
100-178	Ptolemy.